TABLE I. Values of ϵ, β, and n (referred to the cubic axes) for the twelve {111} ⟨110⟩ slip systems.

No. of slip system	Slip plane	Slip direction	$2\epsilon_{xx}$	$2\epsilon_{yy}$	$2\epsilon_{zz}$	$4\epsilon_{yz}$	$4\epsilon_{zx}$	$4\epsilon_{xy}$	$\sqrt{2}\beta_1$	$\sqrt{2}\beta_2$	$\sqrt{2}\beta_3$	$\sqrt{3}n_1$	$\sqrt{3}n_2$	$\sqrt{3}n_3$
1	(111)	$[0\bar{1}1]$	0	$-S_1$	S_1	0	S_1	$-S_1$	0	1	1	1	1	1
2	(111)	$[\bar{1}01]$	$-S_2$	0	S_2	S_2	0	$-S_2$	1	0	1	1	1	1
3	(111)	$[1\bar{1}0]$	S_3	$-S_3$	0	$-S_3$	S_3	0	1	1	0	1	1	1
4	$(11\bar{1})$	[101]	S_4	0	$-S_4$	S_4	0	S_4	1	0	−1	1	1	−1
5	$(11\bar{1})$	[011]	0	S_5	$-S_5$	0	S_5	S_5	0	1	−1	1	1	−1
6	$(11\bar{1})$	$[1\bar{1}0]$	S_6	$-S_6$	0	S_6	$-S_6$	0	1	1	0	1	1	−1
7	$(1\bar{1}1)$	[110]	S_7	$-S_7$	0	S_7	S_7	0	1	−1	0	1	−1	1
8	$(1\bar{1}1)$	$[\bar{1}01]$	$-S_8$	0	S_8	$-S_8$	0	S_8	1	0	1	1	−1	1
9	$(1\bar{1}1)$	[011]	0	$-S_9$	S_9	0	S_9	S_9	0	1	−1	1	−1	1
10	$(\bar{1}11)$	$[0\bar{1}1]$	0	$-S_{10}$	S_{10}	0	$-S_{10}$	S_{10}	0	1	1	−1	1	1
11	$(\bar{1}11)$	[101]	$-S_{11}$	0	S_{11}	S_{11}	0	S_{11}	1	0	−1	−1	1	1
12	$(\bar{1}11)$	[110]	$-S_{12}$	S_{12}	0	S_{12}	S_{12}	0	1	−1	0	−1	1	1

independent (the hydrostatic part being zero) and hence, at least five independent slip systems are required.[11–13] Under highly symmetrical conditions such as exist in the present analysis, however, this requirement can be relaxed.[14]

Values of ϵ_{ij}, together with those of β and n, for the twelve slip systems are given in Table I; cubic axes have been used as reference axes. Values of γ_i have been converted to S_i, the slip density expression in Eqs. (2) and (3). See Appendix for details. Table I essentially follows CSI's notation, with the sense of some slip directions changed so as to conform with the positive direction of the shears as written. Two sign errors in Table III of CSI's paper have also been corrected.

DETAILED CALCULATIONS

1. Wire Drawing

Wire textures of fcc metals and alloys are often a combination of ⟨001⟩ and ⟨111⟩ components; i.e., the grains have their ⟨001⟩ and ⟨111⟩ directions along the wire axis. Hence the effect of wire-drawing of crystals of these two orientations on the slip-induced anisotropy is of interest.

(*a*) *Drawing of a ⟨001⟩ crystal*

Let z be the [001] wire axis, and x and y be the [100] and [010] directions, respectively. The macroscopic strain components are:

$$\epsilon_{xx}=-r/2,\quad \epsilon_{yy}=-r/2,\quad \epsilon_{zz}=r,$$
$$\epsilon_{yz}=\epsilon_{zx}=\epsilon_{xy}=0, \tag{4}$$

where r is the reduction of area. Since wire drawing can be considered as tension along the wire axis,[15] Fig. 1 shows that four of the twelve slip systems, 3, 6, 7, and 12 (see Table I), do not operate because the slip directions are perpendicular to the [001] tensile axis. From Table I, the strain components in terms of the slip densities of the eight active slip systems are:

$$2\epsilon_{xx}=-S_2+S_4-S_8-S_{11},$$
$$2\epsilon_{yy}=-S_1+S_5-S_9-S_{10},$$
$$2\epsilon_{zz}=S_1+S_2-S_4-S_5+S_8+S_9+S_{10}+S_{11}, \tag{5}$$
$$4\epsilon_{yz}=S_2+S_4-S_8+S_{11},$$
$$4\epsilon_{zx}=S_1+S_5+S_9-S_{10},$$
$$4\epsilon_{xy}=-S_1-S_2+S_4+S_5+S_8+S_9+S_{10}+S_{11}.$$

From the symmetry of the slip systems, the $|S_i|$'s must be equal. Then solutions of Eqs. (4) and (5) give

$$S_1=S_2=S_8=S_9=S_{10}=S_{11}=r/4,$$
$$S_4=S_5=-(r/4).^{16} \tag{6}$$

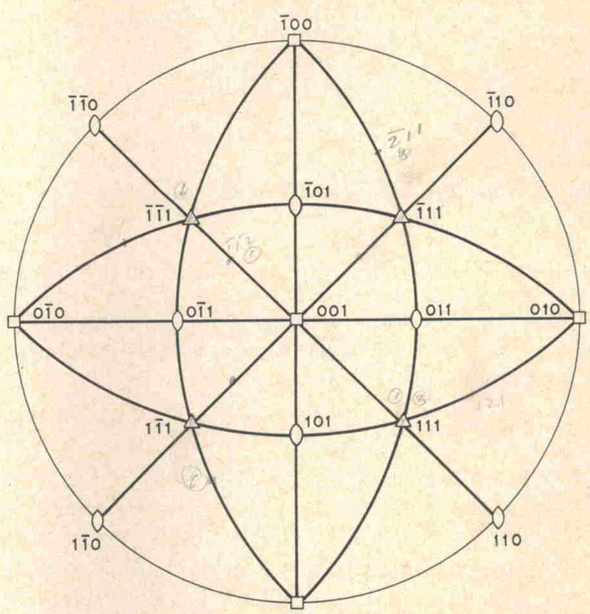

FIG. 1. Standard (001) stereographic projection of cubic crystal.

[14] M. R. Pickus and C. H. Mathewson, J. Inst. Metals **64**, 237 (1939).

[15] Although the stress system in wire-drawing probably consists of a tensile stress σ along the wire axis and a compressive stress $-n\sigma$ along two orthogonal axes in the radial direction, addition of a hydrostatic tension $n\sigma$ (which does not affect slip) will result in the equivalent system of a single tensile stress $(1+n)\sigma$ along the wire axis.

[16] For slip systems (4) and (5), the Miller indices for the slip plane in Table I are the negative of those of Fig. 1, resulting in the sign change of the shears. For simplicity, we shall consistently use the slip density values in Table I for slip systems whose Miller indices for plane or direction are the negative of those listed.

By putting Eq. (6) into (2) and using the pertinent values of β from Table I, one obtains for L.F. deformation

$$E_{\mathrm{LF}}=(1/16)K_{\mathrm{LF}}r(\alpha_1^2+\alpha_2^2+2\alpha_3^2)$$
$$=(1/16)K_{\mathrm{LF}}r\alpha_3^2+\text{const.} \quad (7)$$

Since all values are positive, Eq. (7) means that the induced magnetic energy is a minimum on the x–y plane ($\alpha_3=0$). Hence the wire axis [001] becomes a hard axis of magnetization.

Substitution of Eq. (6) and the n values of Table I into Eq. (3) gives

$$E_{\mathrm{SC}}=(1/16)K_{\mathrm{SC}}[(r/4)\times 0]=0. \quad (8)$$

Hence there is no magnetic anisotropy produced by the S.C. type deformation.

(*b*) *Drawing of a ⟨111⟩ crystal*

Let $x'-[\bar{1}10]$, $y'-[\bar{1}\bar{1}2]$, and $z'-[111]$ be a set of coordinate axes for the macroscopic strain tensor in this system; this retains the z' direction as the wire axis and the other two directions along the radial direction, Fig. 2. The matrix for the transformation of the specimen axes to the cubic axes referred to in Table I is[17]

	x'	y'	z'
X	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{3}}$
Y	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{3}}$
Z	0	$\frac{2}{\sqrt{6}}$	$\frac{1}{\sqrt{3}}$.

Hence from the strain components for wire-drawing [Eq. (4)],

$$\epsilon_{x'x'}=\epsilon_{y'y'}=-r/2,\quad \epsilon_{z'z'}=r,\quad \epsilon_{y'z'}=\epsilon_{z'x'}=\epsilon_{x'y'}=0, \quad (9)$$

and the tensor relation $\epsilon_{ij}=l_{i'i}l_{j'j}\epsilon_{i'j'}\,(i,j=x,\,y,\,z;\ i',j'=x',\,y',\,z')$ [18] where the l's are the components of the transformation matrix, one obtains

$$\epsilon_{xx}=\epsilon_{yy}=\epsilon_{zz}=0,\quad \epsilon_{yz}=\epsilon_{zx}=\epsilon_{xy}=r/2. \quad (10)$$

From the stereographic projection of Fig. 2, the operating slip systems for [111] wire drawing are $(11\bar{1})$ [101], $(11\bar{1})$ [011], $(1\bar{1}1)$ [110], $(1\bar{1}1)$ [011], $(\bar{1}11)$ [101], and $(\bar{1}11)$ [110], corresponding to Nos. 4, 5, 9, 11, and 12 of Table I. The other six systems are inoperative because of zero values of Schmid factor. From Table I, the strain components in terms of slip densities are:

$$2\epsilon_{xx}=S_4+S_7-S_{11}-S_{12},$$
$$2\epsilon_{yy}=S_5-S_7-S_9+S_{12},$$
$$2\epsilon_{zz}=-S_4-S_5+S_9+S_{11}, \quad (11)$$
$$4\epsilon_{yz}=S_4+S_7+S_{11}+S_{12},$$
$$4\epsilon_{zx}=S_5+S_7+S_9+S_{12},$$
$$4\epsilon_{xy}=S_4+S_5+S_9+S_{11}.$$

Solution of Eqs. (10) and (11) gives

$$S_4=S_5=S_7=S_9=S_{11}=S_{12}=r/2. \quad (12)$$

Putting (12) into (2) and using the β values of Table I, one obtains for L.F. deformation

$$E_{\mathrm{LF}}=-\tfrac{1}{8}K_{\mathrm{LF}}r(\alpha_1\alpha_2+\alpha_2\alpha_3+\alpha_3\alpha_1)+\text{const.} \quad (13)$$

Equation (13) is a minimum along [111], or the wire axis. Hence this axis becomes the induced easy axis of magnetization.

Substitution of (12) into (3) and using the n values of Table I gives

$$E_{\mathrm{SC}}=-(1/48)K_{\mathrm{SC}}r(\alpha_1\alpha_2+\alpha_2\alpha_3+\alpha_3\alpha_1). \quad (14)$$

Again the wire axis becomes the induced easy axis.

2. Rolling

Rolling of the following orientations are of interest:

	Roll plane	Roll direction
1.	(001)	$[\bar{1}00]$
2.	(001)	$[\bar{1}10]$
3.	(110)	[001]
4.	(110)	$[\bar{1}12]$
5.	(110)	$[\bar{1}10]$
6.	(111)	$[\bar{1}\bar{1}2]$
7.	(112)	$[\bar{1}10]$
8.	(112)	$[\bar{1}\bar{1}1]$

In all the orientations above, slip directions of the operating slip systems (based on maximum resolved shear stress) are symmetrically disposed about the rolling axis and hence comply with the stability criterion of Pickus and Mathewson.[14] [Orientations (6) and (7), however, were not considered in their discussion of stability.] Tucker[19] has pointed out, however, that the real test of stability is to determine whether slight displacements from a given orientation will cause rotation into, or away from, this orientation. In any case, the following analyses should be valid for rolling reductions in which the orientation does not change significantly. Moreover, by following the orientation change such as with x-ray techniques, the magnetic anisotropy according to the new orientation(s) may still be obtained. Among the above list, (001) $[\bar{1}00]$ is a prominent recrystallization texture in face-centered cubic alloys, while (110) $[\bar{1}12]$ and (112) $[\bar{1}\bar{1}1]$ are often found in the rolled state.[20]

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[17] J. F. Nye, *Physical Properties of Crystals* (Oxford University Press, London, 1960), p. 9.
[18] Ref. 17, p. 11.
[19] G. E. G. Tucker, J. Inst. Metals 82, 655 (1954).
[20] C. S. Barrett, *Structure of Metals* (McGraw-Hill Book Company, Inc., New York, 1952), pp. 484, 509.